

Basic Structure of My Evidential Arguments

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Epistemic Interpretation of Probability

In this [article series](#), when I refer to probability I shall be adopting the *epistemic* interpretation of probability. The epistemic probability of a statement is a measure of the probability that a statement is true, given some stock of knowledge. In other words, epistemic probability measures a person's *degree of belief* in a statement, given some body of evidence. The epistemic probability of a statement can vary from person to person and from time to time (based upon what knowledge a given person had at a given time).¹ For example, the epistemic personal probability that a factory worker Joe will get a pay raise might be different for Joe than it is for Joe's supervisor, due to differences in their knowledge.

Two Types of Evidence and Corresponding Probabilities

Let us divide the evidence relevant to naturalism and theism into two categories. First, certain items of evidence function as "odd" facts that need to be explained. Second, other items of evidence are *background evidence*, which determine the prior probability of rival theories and partially determine their explanatory power.

These two types of evidence have two probabilistic counterparts which are useful for evaluating explanatory hypotheses: (1) the *prior probability* and (2) the *explanatory power* of a hypothesis H. (1) is a measure of how likely H is to occur based on background information B alone, whether or not E is true. As for (2), this measures the ability of a hypothesis (combined with background evidence B) to predict (i.e., make probable) an item of evidence.²

¹ Brian Skyrms, *Choice & Chance: An Introduction to Inductive Logic* (4th ed., Belmont: Wadsworth, 2000), 23.

² I owe these definitions to Robert Greg Cavin in private correspondence.

Notation

Let us proceed, then, to defining some basic probabilistic notation.

B: background information or evidence

E: the evidence to be explained

H: an explanatory hypothesis

R_i: the rival explanatory hypotheses to *H*

Pr(*x*): the probability of *x*

Pr(*x* | *y*): the probability of *x* conditional upon *y*

Next, let us define the following conditional probabilities.

Pr(*H* | *B*) = the *prior probability* of *H* with respect to *B*—a measure of how likely *H* is to occur at all, whether or not *E* is true.

Pr(*R_i* | *B*) = the *prior probability* of *R_i* with respect to *B*—a measure of how likely *R_i* is to occur at all, whether or not *E* is true.

Pr(*E* | *H* & *B*) = the *explanatory power* of *H*—a measure of the degree to which the hypothesis *H* predicts the data *E* given *B*.

Pr(*E* | *R_i* & *B*) = the *explanatory power* of *R_i*—a measure of the degree to which *R_i* predicts *E* given *B*.

Pr(*H* | *E* & *B*) = the *final probability* that *H* is true conditional upon the total evidence *B* and *E*.

Bayes's Theorem

Bayes's Theorem is a mathematical formula which can be used to represent the effect of new information upon our degree of belief in a hypothesis *H*. In its general form, Bayes's Theorem may be expressed as follows:

$$\Pr(\mathbf{H} | B \ \& \ E) = \frac{\Pr(\mathbf{H} | B) \times \Pr(E | B \ \& \ \mathbf{H})}{\Pr(\mathbf{H} | B) \times \Pr(E | B \ \& \ \mathbf{H}) + \sum_{i=1}^n \Pr(A_i | B) \times \Pr(E | B \ \& \ A_i)}$$

Explanatory Arguments

It follows from Bayes's Theorem that *H* will have a high final epistemic probability on the evidence *B* and *E*:

$$\Pr(\mathbf{H} | B \ \& \ E) > 0.5$$

just in case it has a greater overall balance of prior probability and explanatory power than do its alternatives *collectively*:

$$\Pr(\mathbf{H} | B) \times \Pr(E | B \ \& \ \mathbf{H}) > \sum_{i=1}^n \Pr(A_i | B) \times \Pr(E | B \ \& \ A_i)$$

Using this insight enables us to easily define the following argument schema for the explanatory arguments in this series.

Let F be some fact and $H1$ and $H2$ be rival explanatory hypotheses.

1. F is known to be true, i.e., $\Pr(F)$ is close to 1.
2. $H1$ is not intrinsically much more probable than $H2$, i.e., $\Pr(H1 | B)$ is not much more probable than $\Pr(H2 | B)$.
3. $\Pr(F | H2) > \Pr(F | H1)$.
4. Other evidence held equal, $H1$ is probably false, i.e., $\Pr(H1 | B \& F) < 0.5$.³

Evaluating Auxiliary Hypotheses

It is often be useful to determine the evidential significance of an auxiliary hypothesis. In the course of considering various explanatory arguments for naturalism and against theism, we will consider several auxiliary hypotheses. Here are four examples.

- The Evidential Argument from Scale: compares naturalism to theism conjoined with various auxiliary hypotheses about God's potential reasons for giving or not giving humans a privileged position in the universe.
- The Evidential Argument from the Flourishing and Languishing of Sentient Beings: compares naturalism conjoined with Darwinism to theism conjoined with Darwinism
- The Evidential Argument from the Self-Centeredness and Limited Altruism of Human Beings: compares naturalism conjoined with Darwinism to theism conjoined with Darwinism
- The (Evidential) Fine-Tuning Argument: compares theism to naturalism conjoined with the multiverse hypothesis

In the explanatory argument schema described above, it may be possible to defeat premise (3) by showing that an auxiliary hypothesis is evidentially significant in the relevant way, i.e., that A sufficiently raises $\Pr(F | H1)$ or lowers $\Pr(F | H2)$. In order to assess the evidential significance of such hypotheses, we would need to apply the theorem of the probability calculus known as the *theorem of total probability*.

If we let A represent some auxiliary hypothesis, then it follows from the theorem of total probability that:

$$\Pr(F | H1) = \Pr(A | H1) \times \Pr(F | A \& H1) + \Pr(\sim A | H1) \times \Pr(F | H1 \& \sim A)$$

In the context of explanatory arguments, Draper calls that theorem the "weighted average principle" (WAP).⁴ As Draper points out, this formula is an average because $\Pr(A | H1) + \Pr(\sim A | H1) = 1$. It is not a simple straight average, however, since those two values may not equal 1/2; that is why it is a *weighted* average.⁵ The higher $\Pr(A | H1)$, the closer $\Pr(F | H1)$ will be to $\Pr(F | A \& H1)$; similarly, the higher $\Pr(\sim A | H1)$, the closer $\Pr(F | H1)$ will be to $\Pr(F | H1 \& \sim A)$.⁶

³ I owe this schema to Paul Draper.

⁴ Paul Draper, "[Pain and Pleasure: An Evidential Problem for Theists](#)" *Noûs* 23 (1989): 331-50 at 340.

⁵ Draper 1989.

⁶ Draper 1989.

Cumulative Cases

Suppose that F is partitioned into 3 facts, F_1 , F_2 , and F_3 , such that F is logically equivalent to $F_1 \& F_2 \& F_3$. One could easily formulate three *independent* explanatory arguments for H_1 over H_2 , by showing that each item of evidence *individually* is antecedently more probable on H_1 than on H_2 , i.e., $\Pr(F_1 | H_1) > \Pr(F_1 | H_2)$, $\Pr(F_2 | H_1) > \Pr(F_2 | H_2)$, and $\Pr(F_3 | H_1) > \Pr(F_3 | H_2)$. But how could one show that the items of evidence *collectively* are antecedently more probable on H_1 than on H_2 ? In other words, how could one give a *cumulative case* argument?

Using the *chain rule*, it is possible to give a formal, mathematical definition for a cumulative case argument. Using the *chain rule* it follows that :

$$\Pr(F | H \& B) = \Pr(F_1 \& F_2 \& F_3 | H \& B) = \Pr(F_1 | H \& B) \times \Pr(F_2 | F_1 \& H \& B) \times \Pr(F_3 | F_1 \& F_2 \& H \& B)$$

Therefore, F_1 , F_2 , and F_3 are a cumulative case for H_1 and against H_2 just in case:

$$\Pr(F | H_1 \& B) = \Pr(F_1 \& F_2 \& F_3 | H_1 \& B) = \Pr(F_1 | H_1 \& B) \times \Pr(F_2 | F_1 \& H_1 \& B) \times \Pr(F_3 | F_1 \& F_2 \& H_1 \& B)$$

is greater than:

$$\Pr(F | H_2 \& B) = \Pr(F_1 \& F_2 \& F_3 | H_2 \& B) = \Pr(F_1 | H_2 \& B) \times \Pr(F_2 | F_1 \& H_2 \& B) \times \Pr(F_3 | F_1 \& F_2 \& H_2 \& B)$$

Another way to compare these two values is to compare the *ratio* of H_1 's final probability to H_2 's final probability. The ratio of these two formulas gives us Bayes's Theorem in its compound odds form:

$$\frac{\Pr(H_2 | B \& F)}{\Pr(H_1 | B \& F)} = \frac{\Pr(H_2 | B)}{\Pr(H_1 | B)} \times \frac{\Pr(F | B \& H_2)}{\Pr(F | B \& H_1)}$$

If this ratio is greater than 1, then H_1 's final probability is greater than H_2 's final probability; if the ratio is less than 1, then *vice versa*; if the ratio equals one, then H_1 and H_2 have equal final probabilities. This insight gives us one way to show that one hypothesis has a higher final probability than the other hypothesis. For example, suppose want to show that H_2 has a higher final probability than H_1 . In other words, we want to show that:

$$\Pr(H_2 | B \& F) > \Pr(H_1 | B \& F)$$

One way to show that $H2$ has a higher final probability than $H1$ is to show that the ratio of each multiplicand on the right-hand side of Bayes's Theorem in its compound odds form is also less than 1, i.e.,

$$w: \frac{\Pr(H2 | B)}{\Pr(H1 | B)} > 1$$

and

$$x: \frac{\Pr(F1 | H2 \& B)}{\Pr(F1 | H1 \& B)} > 1$$

and

$$y: \frac{\Pr(F2 | F1 \& H2 \& B)}{\Pr(F2 | F1 \& H1 \& B)} > 1$$

and

$$z: \frac{\Pr(F3 | F2 \& F1 \& H2 \& B)}{\Pr(F3 | F2 \& F1 \& H1 \& B)} > 1.$$

This enables us to define the following schema for cumulative case arguments.

1. $\Pr(F1 | H2 \& B) > \Pr(F1 | H1 \& B)$. [from formula x]
2. $\Pr(F2 | F1 \& H2 \& B) > \Pr(F2 | F1 \& H1 \& B)$. [from formula y]
3. $\Pr(F3 | F1 \& F2 \& H2 \& B) > \Pr(F3 | F1 \& F2 \& H1 \& B)$. [from formula z]
4. Some fact F is known to be true, i.e., $\Pr(F | B)$ is close to 1.
5. $H2$ is intrinsically more probable than $H1$, i.e., $\Pr(H2 | B) > \Pr(H1 | B)$. [from formula w]
6. Other evidence held equal, $H1$ is probably false, i.e., $\Pr(H1 | B \& E) < 0.5$. [By Bayes's Theorem]

This schema can obviously be expanded as needed to incorporate the desired number of cumulative lines of evidence.